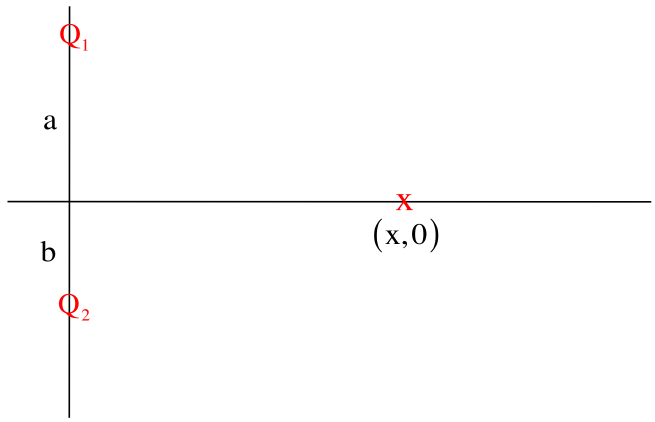
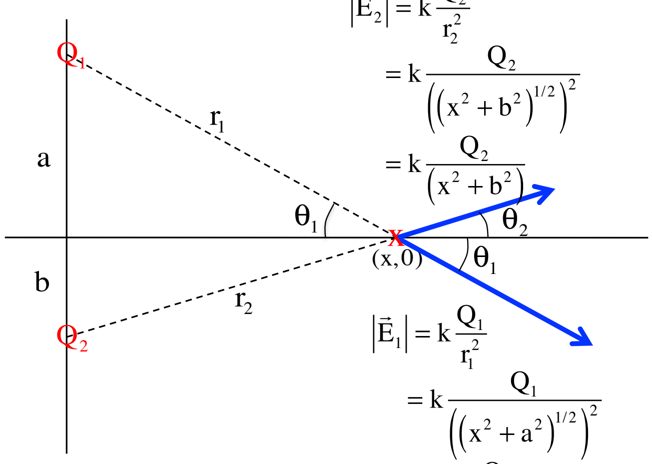
A.) For the point charge configuration shown, derive the expression for the electric field as it exists at (x,0).



The electric field magnitudes due to the two charges:



$$\left|\vec{E}_2\right| = k\frac{Q_2}{r_2^2} = k\frac{Q_2}{\left(\left(x^2+b^2\right)^{1/2}\right)^2} = k\frac{Q_2}{\left(x^2+b^2\right)}$$

$$\left|\vec{E}_1\right| = k\frac{Q_1}{r_1^2} = k\frac{Q_1}{\left(\left(x^2+a^2\right)^{1/2}\right)^2} = k\frac{Q_1}{\left(x^2+a^2\right)}$$

Note that the trig functions in terms of length parameters that were given are:

$$\sin\theta_1 = \frac{a}{\left(x^2+a^2\right)^{1/2}}$$

$$\cos\theta_1 = \frac{x}{\left(x^2+a^2\right)^{1/2}}$$

$$\sin\theta_2 = \frac{b}{\left(x^2+b^2\right)^{1/2}}$$

$$\cos\theta_2 = \frac{x}{\left(x^2+b^2\right)^{1/2}}$$

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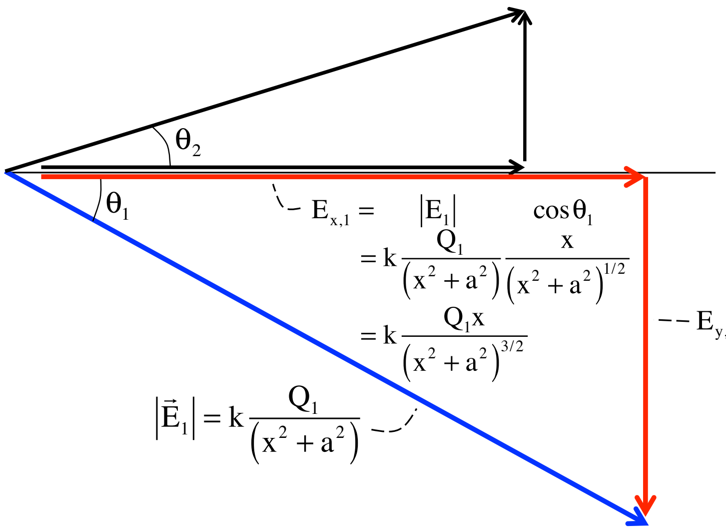
With that, the x and y component of E_1 are:



$$E_{x,1} = |E_1| \cos\theta_1 = k\frac{Q_1}{\left(x^2+a^2\right)}\frac{x}{\left(x^2+a^2\right)^{1/2}} = k\frac{Q_1 x}{\left(x^2+a^2\right)^{3/2}}$$

$$E_{y,1} = |E_1| \sin\theta_1 = k\frac{Q_1}{\left(x^2+a^2\right)}\frac{a}{\left(x^2+a^2\right)^{1/2}} = k\frac{Q_1 a}{\left(x^2+a^2\right)^{3/2}}$$

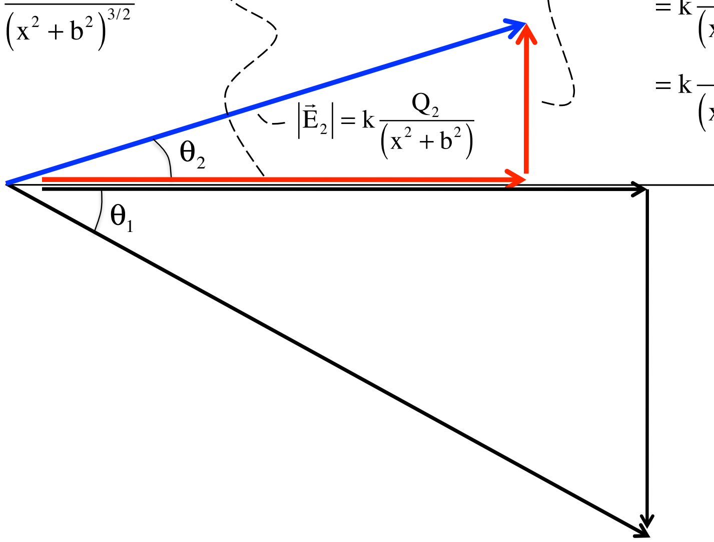
$$\left|\vec{E}_1\right| = k\frac{Q_1}{\left(x^2+a^2\right)}$$

Likewise, the x and y component of E_2:

$$E_{x,2} = |E_2| \cos\theta_2 = k\frac{Q_2}{\left(x^2+b^2\right)}\frac{x}{\left(x^2+b^2\right)^{1/2}} = k\frac{Q_2 x}{\left(x^2+b^2\right)^{3/2}}$$

$$E_{y,2} = |E_2| \sin\theta_2 = k\frac{Q_2}{\left(x^2+b^2\right)}\frac{b}{\left(x^2+b^2\right)^{1/2}} = k\frac{Q_2 b}{\left(x^2+b^2\right)^{3/2}}$$

$$\left|\vec{E}_2\right| = k\frac{Q_2}{\left(x^2+b^2\right)}$$



Putting it all together, we get:

$$\vec{E} = \left(|E_2|\cos\theta_2 + |E_1|\cos\theta_1 \right)\hat{i} + \left(|E_2|\sin\theta_2 - |E_1|\sin\theta_1 \right)\hat{j}$$

$$= \left(k\frac{Q_2 x}{\left(x^2+b^2\right)^{3/2}} + k\frac{Q_1 x}{\left(x^2+a^2\right)^{3/2}} \right)\hat{i} + \left(k\frac{Q_2 b}{\left(x^2+b^2\right)^{3/2}} - k\frac{Q_1 a}{\left(x^2+a^2\right)^{3/2}} \right)\hat{j}$$